Problem Set III: Due February 4, 2015

1.)   a.)   Consider a weakly damped linear harmonic oscillator driven by white noise.

         i.)   Derive the fluctuation spectrum at thermal equilibrium.

         ii.)   What value of forcing is required to achieve stationarity at temperature T?

    b.)   Now consider a forced nonlinear oscillator

$$\ddot{x} + \gamma\dot{x} + \omega_0^2 x + \alpha x^3 = \tilde{f}.$$

         Again, assume $\tilde{f}$ is white noise. Characterize the equilibrium fluctuation spectrum as completely as you can. Hint: You may find it useful to review Section 29 of "Mechanics", by Landau and Lifshitz.

2.)   Consider a time-dependent phenomenological Ginzburg-Landau system, which satisfies

$$\frac{\partial \tilde{\eta}}{\partial t} - \Gamma_0 a \nabla^2 \tilde{\eta} = -\frac{\widetilde{\delta H}}{\delta \eta} + \tilde{f},$$

$$H = +b(T - T_c)\frac{\eta^2}{2} + \frac{c\eta^4}{4},$$

    where $\tilde{f}$ is white noise, in both space and time. Here $\tilde{\eta}$ is a fluctuation and η_0 is the mean value of the order parameter.

    a.)   Using the linear response, calculate the fluctuation spectrum and intensity for $\langle \tilde{\eta}^2 \rangle$. Assume $T > T_c$, then consider $T < T_c$.

How does your result behave as:

  i.) $k \rightarrow 0$ (large scale)

  ii.) $T \rightarrow T_c$, from above (near criticality)

b.)   Estimate when your calculation in a.) breaks down. Congratulations! -
        You have just re-derived the Ginzburg Criterion.

3.)   a.)   Derive the dispersion relation for a simple acoustic wave.

        b.)   Derive an energy theorem for the acoustic wave. Your theorem should
                have a structure similar to the Poynting theorem in Electromagnetism.

        c.)   Now derive the dispersing relation for an ion acoustic wave. Take
                $v_{Thi} \ll w/k < v_{The}$. Take $T_c \rightarrow 0$. Use a fluid model.

4.)   a.)   Compute the average power dissipated by a test particle moving at
                velocity $\underline{v}$ thru a plasma.   Do this by computing $\langle \underline{E} \cdot \underline{J} \rangle$,
                for $\underline{J} = q\delta(\underline{x} - \underline{v}t)$.

        b.)   Now, integrate your result from a.) over a distribution $\langle f \rangle$ of test
                particles and compare that to the power dissipated by the dynamical
                friction term in the Lenard-Balescu equation.

        c.)   What can we conclude from this? Discuss your result.

5.)   Kulsrud 10.1

6.)   Kulsrud 10.2

7.)   Kulsrud 10.3